At the end of Packet #1we worked on multiplying monomials, binomials, and trinomials. What we have to learn now is how to "go backwards" and do what is called *factoring*.

The two meanings of Factor

1. Factor (verb) : To rewrite an algebraic expression as an **equivalent product**

2. Factor (noun) : An algebraic expression that is one part of a larger factored expression.

We have a few different methods of factoring and we need to be great at all of them. The 1st method is the factoring with the greatest common factor or GCF.

Factoring using GCF:

Take the greatest common factor (GCF) for the numerical coefficient. When choosing the GCF for the variables, if all the terms have a common variable, take the one with the lowest exponent.

ie) $9x^4 + 3x^3 + 12x^2$

GCF: coefficients: 3

Variable (x) : x^2

GCF: $3x^2$

What's left? Division of monomials:

$9x^4/3x^2$ $3x^3/3x^2$ $12x^2/3x^2$

$3x^2$ x 4

Factored Completely: $3x^2(3x^2 + x + 4)$

*Make sure you keep the GCF as part of your final answer.

Factor each problem using the GCF and check by distributing:

1) $14x^9 - 7x^7 + 21x^5$

2) $26x^4y - 39x^3y^2 + 52x^2y^3 - 13xy^4$

3) $32x^6 - 12x^5 - 16x^4$

4) $16x^5y^2 - 8x^4y^3 + 24x^2y^4 - 32xy^5$

5) $24b^{11} + 4b^{10} - 6b^9 + 2b^8$

6) $96a^5b + 48a^3b^3 - 144ab^5$

Exercise #1: Consider the expression $6x^2 + 15x$

(a)Write the individual terms $6x^2$ and $15x$ as completely factored expressions. Determine their **greatest common factor (GCF).**

(b) Using the Distributive Property, rewrite $6x^2 + 15x$ as a product involving the **GCF** from part (a).

(c) Evaluate both $6x^2 + 15x$ and your answer from part (b) for $x = 2$. What did you find? What does this support about the two expressions?

It is important that you are **fluent in reversing the distributive property** through using the greatest common factor **(GCF)**. The coefficient component of the GCF is the largest number that goes into all the terms evenly. *If all the terms have a common variable, the GCF is the variable with the lowest exponent.*

Exercise #2: Write each polynomial below as a factored expression involving the GCF of the polynomial. You then check your answer by distributing the GCF.

a) $6x^2 + 10x$

(b) $3x - 24$

(c) $10x^2 - 15x$

$2x(3x + 5)$

Check:
$2x(3x + 5)$
$6x^2 + 10x$

(d) $4x^2 + 8x + 24$

(e) $6x^3 - 8x^2 + 2x$

(f) $10x^3 - 35x^2$

(g) $10x^2 - 40x - 50$

(h) $10x^4 - 2x^2$

(i) $8x^3 + 24x^2 - 32x$

Being able to **fluently factor out a GCF** is an essential skill. Sometimes a GCF is more than just a monomial. We have done this type of factoring back in Unit #1.

Exercise #3: Rewrite each expression as a product of two binomials by factoring out a common binomial factor.

(a) $(x+5)(x-1) + (x+5)(2x-3)$

(b) $(2x-1)(2x+7) – (2x-1)(x-3)$ ***Be careful w subtraction!***

(c) $(x+9)(x+7) + (x+7)(2x+5)$

(d) $(5x-4)(3x+7) – (x-1)(5x-4)$

Exercise #4: Applications of the GCF and Common Core type questions:

The area of a rectangle is represented by $16x^2 + 56x$.

The width of the rectangle is given as $2x + 7$.

(a) Give a monomial expression in terms of x for your length of the rectangle. Show you arrived at how you arrived at your answer.

(b) If the length of the rectangle is 80, what is the width of the rectangle. Explain how your answer.

Q1 Quiz 6 Review

Factor each problem using the GCF and check by distributing:

1) $36x^9 - 60x^7 + 18x^5$

2) $45x^4y - 54x^3y^2 + 276x^2y^3 - 9xy$

3) $60x^6 - 80x^5 - 50x^4$

4) $80x^5y^2 - 64x^4y^3 + 32x^2y^4 - 48xy^5$

5) $144b^{11} + 72b^{10} - 120b^9 + 24b^8$

6) $42a^5b + 63a^3b^3 - 28ab^5$

Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor.

7) $(x+7)(x-1) + (x+7)(2x-3)$

8) $(4x-1)(5x+7) - (4x-1)(7x-3)$

9) $9x(x+1) + 7(x+1)$

10) $4x(x+6) + 9(x+6)$

11.) The area of a rectangle is represented by the polynomial $24x^2 + 60x$. The width of the rectangle is given by the binomial $2x + 5$.

(a) Given a monomial expression in terms of x for the length of the rectangle. Show how you arrived at your answer.

(b) If the length of the rectangle is 80, what is the width of the rectangle? Explain your thinking.

12.) The area of a rectangle is represented by the polynomial $42x^2 + 98x$. The width of the rectangle is given by the binomial $3x + 7$.

(a) Given a monomial expression in terms of x for the length of the rectangle. Show how you arrived at your answer.

(b) If the length of the rectangle is 80, what is the width of the rectangle? Explain your thinking.

HOW TO FACTOR TRINOMIALS

Remember your hints:

A. When the last sign is addition

$x^2 - 5x + 6$ 1)Both signs the same

(x)(x)

2) Factors of +6 w/ a sum of -5. (-3 and -2)

(x - 3)(x - 2)

B. When the last sign is subtraction

$x^2 + 5x - 36$ 1) signs are different

(x -)(x +)

2) Factors of -36 w/ a sum of +5 (+9 and -4)

(x - 4)(x + 9)

Factor each trinomial into two binomials check by using FOIL:

1) $x^2 + 7x + 6$

2) $x^2 - 8x + 12$

3) $x^2 - 10x + 16$

4) $x^2 + 4x - 21$

5) $x^2 - 8x - 33$

6) $x^2 + 5x - 6$

7) $x^2 + 16x + 64$

8) $x^2 + 11x - 26$

9) $x^2 - 12x + 27$

10) $x^2 - 17x + 72$

11) $x^2 + 6x - 72$

12) $x^2 + 5x - 66$

13) $x^2 - 17x + 52$

14) $x^2 - 22x + 121$

15) $x^2 + 8x + 16$

16) $x^2 + 6x - 7$

17) $x^2 - 11x - 42$

18) $x^2 + 24x + 144$

19) $x^2 + 2x - 35$

20) $x^2 - 5x - 66$

21) $x^2 - 14x + 48$

22) $x^2 + x - 42$

23) $x^2 + x - 56$

24) $x^2 - 14x + 45$

25) $x^2 + 15x + 36$

26) $x^2 + 7x - 18$

27) $x^2 + 10x - 24$

28) $x^2 + 14x + 24$

29) $x^2 + 29x + 28$

30) $x^2 - 3x - 18$

31) $x^2 - 9$

32) $x^2 - 36$

33) $x^2 - 121$

34) $9x^2 - 25$

35) $144x^2 - 49$

36) $64x^2 - 81$

37) $x^2 + 100$

38) $x^2 - 44$

39) $x^2 - x - 9$

1) (x+6)(x+1)
2) (x-6)(x-2)
3) (x-8)(x-2)
4) (x+7)(x-3)
5) (x+3)(x-11)
6) (x+6)(x-1)
7) (x+8)(x+8)
8) (x+13)(x-2)
9) (x-9)(x-3)
10) (x-8)(x-9)
11) (x+12)(x-6)
12) (x+11)(x-6)
13) (x-13)(x-4)
14) (x-11)(x-11)
15) (x+4)(x+4)
16) (x+7)(x-1)
17) (x+3)(x-14)
18) (x+12)(x+12)
19) (x+7)(x-5)
20) (x-11)(x+6)
21) (x-8)(x-6)
22) (x+7)(x-6)
23) (x+8)(x-7)
24) (x-9)(x-5)
25) (x + 12)(x + 3)
26) (x + 9)(x - 2)
27) (x + 12)(x - 2)
28) (x+2)(x+12)
29) (x + 28)(x + 1)
30) (x + 3)(x - 6)
31) (x + 3)(x -3)
32)(x + 6)(x – 6)
33) (x + 11)(x – 11)
34) (3x + 5)(3x – 5)
35) (12x +7)(12x – 7)
36) (8x + 9)(8x – 9)
37) Prime (SOTS not DOTS)
38) Prime (44 is not a perfect square)
39) Prime (No f of 9 w/ a diff = 1)

Q1 Quiz 7 Review

1) $x^2 + 7x + 6$

2) $x^2 - 8x + 12$

3) $x^2 - 10x + 16$

4) $x^2 + 4x - 21$

5) $x^2 - 8x - 33$

6) $x^2 + 5x – 6$

7) $x^2 + 16x + 64$

8) $x^2 + 11x - 26$

9) $x^2 - 12x + 27$

10) $x^2 - 17x + 72$

11) $x^2 + 6x - 72$

12) $x^2 + 5x – 66$

13) $x^2 - 17x + 52$

14) $x^2 - 22x + 121$

15) $x^2 + 8x + 16$

16) $x^2 + 6x - 7$

17) $x^2 - 11x - 42$

18) $x^2 + 24x + 144$

19) $x^2 + 2x - 35$

20) $x^2 - 5x - 66$

21) $x^2 - 14x + 48$

22) $x^2 - 100$

23) $x^2 - 49$

24) $x^2 - 256$

25) $4x^2 - 121$

26) $16x^2 - 81$

27) $225x^2 - 1$

28) $64x^2 - 169$

29) $9x^2 - 289$

30) $324x^2 - 361$

Answer Key:

1) (x+6)(x+1)
2) (x-6)(x-2)
3) (x-8)(x-2)
4) (x+7)(x-3)
5) (x+3)(x-11)
6) (x+6)(x-1)
7) (x+8)(x+8)
8) (x+13)(x-2)
9) (x-9)(x-3)
10) (x-8)(x-9)
11) (x+12)(x-6)
12) (x+11)(x-6)
13) (x-13)(x-4)
14) (x-11)(x-11)
15) (x+4)(x+4)
16) (x+7)(x-1)
17) (x+3)(x-14)
18) (x+12)(x+12)
19) (x+7)(x-5)
20) (x-11)(x+6)
21) (x-8)(x-6)
22) (x+10)(x-10)
23) (x+7)(x-7)
24) (x+16)(x-16)
25) (2x+11)(2x-11)
26) (4x+9)(4x-9)
27) (15x+1)(15x-1)
28) (8x+13)(8x-13)
29) (2x+17)(2x-17)
30) (18x+19)(18x-19)

2Step Factoring with a GCF:

$6x^2 - 6x - 72$ $8x^7 + 88x^6 + 240x^5$ $3x^2 - 108$

Step 1: Take out the GCF

$6(x^2 - x - 12)$ $8x^5(x^2 + 11x + 30)$ $3(x^2 - 36)$

Step 2: Factor what's left (if possible) using your factoring rules:

$6(x+3)(x-4)$ $8x(x+6)(x+5)$ $3(x+6)(x-6)$

Factor using GCF and then factor the trinomial (then check):

40) $4x^2 + 20x + 24$ 41) $10x^2 - 80x + 150$ 42) $9x^2 + 90x - 99$

43) $3x^3 + 27x^2 + 60x$ 44) $12x^6 + 72x^5 + 60x^4$ 45) $8x^9 + 40x^8 - 192x^7$

46) $12x^2 - 12$ 47) $25x^2 - 100$ 48) $5x^5 - 320x^3$

Answer Key:

40) $4(x+2)(x+3)$ *41) $10(x-5)(x-3)$* *42) $9(x+11)(x-1)$*
43) $3x(x+4)(x+5)$ *44) $12x^4(x+5)(x+1)$* *45) $8x^7(x+8)(x-3)$*
46) $12(x+1)(x-1)$ *47) $25(x+2)(x-2)$* *48) $5x^3(x+8)(x-8)$*

Q1 Quiz 8 Review

2Step Factoring with a GCF:

Examples:

$6x^2 - 6x - 72$ $8x^7 - 48x^6 + 40x^5$ $27x^2 - 75$

Step 1: Take out the GCF

Take the greatest common factor (GCF) for the numerical coefficient. When choosing the GCF for the variables, if all the terms have a common variable, take the one with the lowest exponent

$6(x^2 - x - 12)$ $8x^5(x^2 - 6x + 5)$ $3(9x^2 - 25)$

Step 2: Factor what's left (if possible) using your factoring rules:

6(x – 4)(x + 3) ***$8x^5$(x - 1)(x - 5)*** ***3(3x + 5)(3x – 5)***

****Don't forget to include your GCF in your final answer as it is a FACTOR.***

Practice Problems:

1) $12x^2 - 3$ 2) $6x^2 - 6x - 432$ 3) $18x^2 + 54x - 972$

4) $8x^8 + 80x^7 - 88x^6$ 5) $10x^4 - 120x^3 + 360x^2$ 6) $16x^2 - 144$

7) $20x^2 - 245$ 8) $64x^9 - 100x^7$ 9) $225x^8 - 9x^6$

10) $7x^2 + 84x + 245$

11) $12x^5 – 48x^4 – 540x^3$

12) $9x^2 + 99x - 108$

13) $6x^2 - 60x + 126$

14) $7x^6 – 112x^5 + 105x^4$

15) $16x^9 + 16x^8 – 320x^7$

16) $144x^2 – 4$

17) $100x^2 – 25$

18) $100x^2 – 225$

Answer Key:

1) $3(2x+1)(2x-1)$
2) $6(x+8)(x-9)$
3) $19(x+9)(x-6)$
4) $8x^6(x+11)(x-1)$
5) $10x^2(x-6)(x-6)$
6) $6(x+3)(x-3)$
7) $5(2x+7)(2x-7)$
8) $4x^7(4x+5)(4x-5)$
9) $9x^6(5x+1)(5x-1)$
10) $7(x+5)(x+7)$
11) $12x^3(x+5)(x-9)$
12) $9(x+12)(x-1)$
13) $6(x-7)(x-3)$
14) $7x^4(x-15)(x-1)$
15) $16x^7(x+5)(x-4)$
16) $4(6x+1)(6x-1)$
17) $25(2x+1)(2x-1)$
18) $25(2x+3)(2x-3)$

Case II Factoring (a ≠ 1)

Factoring a trinomial with a coefficient for x^2 other than 1

Factor: $6x^2 + 5x - 4$

1) Look for a GCF:
   a. There is no GCF for this trinomial
   b. The only way this method works is if you take out the GCF (if there is one.)
2) Take the coefficient for x^2 (6) and multiply it with the last term (4):
   $6x^2 + 5x - 4$ $\quad 6 \cdot 4 = 24$
   * Now find factors of 24 with a difference of 5
   8 and 3 [with the 8 going to the + (+5x)]
   $6x^2 + 8x - 3x - 4$
3) **SPLIT THE MIDDLE** and reduce each side:

   $6x^2 + 8x \mid - 3x - 4$ $\quad$ *When you're done the binomial on each side should be the same.
   Take Out: 2x and -1
   $2x(3x + 4) - 1(3x + 4)$

4) Your binomial factors are (2x -1) and (3x + 4)

   $(2x - 1)(3x + 4)$

5) **FOIL CHECK**

   $(2x - 1)(3x + 4)$
   $6x^2 - 8x + 3x - 4$

   $6x^2 + 5x - 4$

Extra Problems: (Remember... GCF 1st)

1) $7x^2 + 19x - 6$ $\quad$ 2) $36x^2 - 21x + 3$

3) $12x^2 - 16x + 5$

4) $20x^2 + 42x - 20$

5) $9x^2 - 3x - 42$

6) $16x^2 - 10x + 1$

7) $24x^2 + x - 3$

8) $9x^2 + 35x - 4$

9) $16x^2 + 8x + 1$

10) $48x^2 + 16x - 20$

Q1 Quiz 9 Review:

1) $6x^2 + 5x - 6$

2) $6x^2 - 9x - 6$

3) $12x^5 - 20x^4 + 7x^3$

4) $12x^2 + 22x + 6$

5) $8x^2 – 9x -14$

6) $8x^2 + 20x – 12$

7) $32x^2 – 4x – 10$

8) $32x^2 + 12x – 2$

9) $24x^2 + 27x + 3$

10) $24x^2 – 17x + 3$

11) $60x^2 + 24x - 3$

12) $60x^2 – 88x - 3$

Answer Key:

1) (2x+3)(3x-2) ***2) 2(2x+1)(x-2)*** ***3) x^3(2x-1)(6x-7)*** ***4) 2(2x+3)(3x+1)***
5) (8x+7)(x-2) ***6) 4(x+3)(2x-1)*** ***7) 2(2x+1)(8x-5)*** ***8) 2(2x+1)(8x-1)***
9) 3(x+1)(8x+1) ***10) (8x-3)(3x-1)*** ***11) 3(2x+1)(10x-1)*** ***12) (30x+1)(2x-3)***

Mixed Review:

Factor using the GCF:

1) $16x^5y^2 - 8x^4y^3 + 24x^2y^4 - 32xy^5$

2) $24b^{11} + 4b^{10} - 6b^9 + 2b^8$

3) $(4x-1)(5x+7) - (2x-8)(5x+7)$

4) $9x(3x-5) + 7(3x-5)$

5) The area of a rectangle is represented by the polynomial $27x^2 + 90x$. The width of the rectangle is given by the binomial $3x + 5$.

(a) Given a monomial expression in terms of x for the length of the rectangle. Show how you arrived at your answer.

(b) If the length of the rectangle is 63, what is the width of the rectangle? Explain your thinking.

Factor using Case I rules

6) $x^2 - 14x + 48$ 7) $x^2 - 3x - 54$ 8) $x^2 + 2x - 80$ 9) $x^2 + 17x + 66$

10) $x^2 - 14x - 15$ 11) $x^2 + 4x - 96$ 12) $x^2 + 22x + 121$ 13) $x^2 - 17x + 66$

Factor using 2Step rules:

14) $3x^2 - 18x - 120$ 15) $8x^2 + 8x - 336$ 16) $6x^2 - 84x + 198$

17) $7x^2 - 252$ 18) $81x^2 - 9$ 19) $81x^2 - 144$

Factor using Case II rules (remember: look for GCF 1st):

20) $3x^2 - 16x + 13$ 21) $14x^2 + 8x - 6$ 22) $14x^2 + x - 3$

"Case II Mix":

Factor each quadratic. Remember to always look for a GCF 1st. Then decide what type of factoring problem it is. It might be Case I or it might be Case II (grouping). It can even be DOTS. Good luck!

1) $20x^2 + 14x - 6$

2) $20x^2 - 60x - 800$

3) $20x^2 - 19x + 3$

4) $24x^2 + 18x + 3$

5) $24x^2 + 7x - 6$

6) $24x^2 - 168x - 1{,}056$

7) $36x^2 - 3x - 3$

8) $36x^2 - 36x - 7$

9) $32x^2 + 160x - 768$

10) $100x^2 - 9$

11) $100x^2 - 25$

12) $400x^2 - 225$

Answer Key: 1) $2(x+1)(10x-3)$ 2) $20(x+5)(x-8)$ 3) $(4x-3)(5x-1)$
4) $3(2x+1)(4x+1)$ 5) $(3x+2)(8x-3)$ 6) $24(x+4)(x-11)$
7) $3(3x-1)(4x+1)$ 8) $(6x-7)(6x+1)$ 9) $32(x+8)(x-3)$
10) $(10x+3)(10x-3)$ 11) $25(2x+1)(2x-1)$ 12) $25(4x+3)(4x-3)$

Factor each trinomial and FOIL Check:

1) $x^2 - 6x - 72$

2) $x^2 + 14x + 13$

3) $x^2 - 19x + 88$

4) $x^2 + 2x - 63$

5) $x^2 - 196$

6) $x^2 - 1$

7) $x^2 + 20x + 64$

8) $x^2 + 11x - 12$

9) $x^2 - 12x + 35$

10) $x^2 - 17x + 70$

11) $x^2 + 14x - 72$

12) $x^2 + 5x - 36$

13) $x^2 - 20x + 96$

14) $x^2 - 24x + 144$

15) $x^2 + 10x + 25$

Factor using the GCF (check by distributing):

16) $24x^{10} - 144x^{9} + 48x^{8}$

17) $64x^5y^3 – 40x^4y^4 + 32x^3y^4 – 8x^2y^3$

Factor using the GCF and then factor the quadratic and check:

18) $x^4 – 15x^3 + 56x^2$

19) $4x^2 + 24x – 240$

20) $5x^3 – 5x^2 –360x$

21) $12x^2 – 243$

22) $16x^2 – 16$

23) $8x^{17} – 512x^{15}$

Mixed Problems:

24) $49x^2 - 25$

25) $4x^2 - 121$

26) $x^4 - 36$

27) $x^{16} - 64$

28) $x^{100} - 169$

29) $48x^8 - 12$

30) $25x^2 - 100$

31) $36x^4 - 9$

32) $100x^2 - 225$

33) $x^2 + 64$

34) $x^2 - 48$

35) $x^2 - 2x + 24$

36) $x^2 + 11x - 30$

37) $5x^2 + 20$

38) $7x^2 - 7x - 84$

Factoring with x^4

Factor: $x^4 + 5x^2 - 24$

1) Look for a GCF:
   a. There is no GCF for this trinomial
   b. The only way this method works is if you take out the GCF (if there is one.)
2) Take the coefficient for x^2 (1) and multiply it with the last term (24):

$x^4 + 5x^2 - 24$

Since the coefficient for x^4 is 1, factor using your case I rules.
But instead of starting with (x)(x), you start with
$(x^2 \quad)(x^2 \quad)$ since x^2 times x^2 equals x^4

$(x^2 \quad)(x^2 \quad)$

* Now find factors of -24 with a sum of +5. The numbers will be +8 and -3.

$(x^2 + 8)(x^2 - 3)$

3) Check: $(x^2 + 8)(x^2 - 3)$
$x^2 (x^2 - 3) + 8(x^2 - 3)$
$x^4 - 3x^2 + 8x^2 - 24$
$x^4 + 5x^2 - 24$ (It checks!!)

1) $x^4 + 13x^2 + 22$

2) $x^4 - 5x^2 - 66$

3) $x^4 + x^2 - 156$

4) $x^4 - 25x^2 + 24$

5) $x^4 - 12x^2 + 27$

6) $x^4 - 25$

7) $9x^4 - 16$

8) $121x^4 - 4$

9) $x^4 - 81$

10) $x^4 - 1$

11) $x^4 - 13x^2 + 36$

12) $x^4 - 17x^2 + 16$

13) $x^4 + 3x^2 - 4$

14) $81x^4 - 1$

Factoring with x^4 (Case II)

Factor: $2x^4 + 13x^2 - 24$

1) Look for a GCF:
   a. There is no GCF for this trinomial
   b. The only way this method works is if you take out the GCF (if there is one.)
2) Take the coefficient for x^2 (1) and multiply it with the last term (24):

$2x^4 + 13x^2 - 24$ — Since the coefficient for x^4 is 2 (not 1), you need to follow your CASE II (grouping) rules.

$2 \cdot (-24) = -48$

$2x^4 + 16x^2 - 3x^2 - 24$ — * Now find factors of -48 with a sum of +13. The numbers will be +16 and -3, which become +16x and -3x.

3) **SPLIT THE MIDDLE** and reduce each side:

$2x^4 + 16x^2 \mid - 3x^2 - 24$ — *When you're done the binomial on each side should be the same.

Take Out: x and -8

$2x^2 (x^2 + 8) - 3(x^2 + 8)$

4) Take out the common binomial $(x^2 + 8)$ as a GCF, and you are left with $2x^2$ on the left and – 3 on the right. They make up the binomial $(2x^2 - 3)$

5) Your binomial factors are $(x^2 + 8)$ and $(2x^2 - 3)$

6) Check: $(x^2 + 8)(2x^2 - 3)$

$x^2 (2x^2 - 3) + 8(2x^2 - 3)$

$2x^4 - 3x^2 + 16x^2 - 24$

$2x^4 + 13x^2 - 24$ (It checks!!)

1) $5x^4 + 29x^2 + 20$

2) $3x^4 - 7x^2 - 6$

3) $8x^4 + x^2 - 7$

4) $3x^4 - 22x^2 + 24$

5) $10x^4 - 13x^2 + 4$

6) $12x^4 - 7x^2 - 5$

7) $9x^4 - 121$

8) $49x^4 - 4$

9) $x^4 - 81$

10) $16x^4 - 81$

11) $25x^4 - 196$

12) $324x^4 - 1$